tion of the relative geographical distribution of Io and Angitrema, taken in connection with obvious resemblances in the shells detected by Mr. Reeve, seem to favor the suggestion that Angitrema is but a minor phase of Io. In habits the animals, so far as is known, are somewhat similar to each other, with this difference, that Angitrema is fitted to dwell in more quiet waters than the necessities of Io require.

Taken in another aspect, the inquiry why Io should be confined apparently to the head-waters of the Tennessee River can be answered (in the proverbial Yankee style) by offsetting the inquiry why a curious group of shells with a fissured lip should be found only in the Coosa River in Alabama. This, like much more that might be made the subject of inquiry, is a part of the unwritten history of Io that remains to be investigated.

The reader who may desire to refer to a summary of what has been written on species of Io will find such information as is available for the purposes of a naturalist in a work entitled Strepomatidæ, by G. W. Tryon, Jr., Smithsonian Miscellaneous Collections, No. 253.

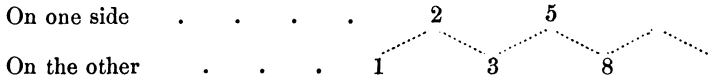
A POPULAR EXPLANATION (FOR THOSE WHO UNDERSTAND BOTANY) OF THE MATHEMATICAL NATURE OF PHYLLOTAXIS.

BY THE LATE CHAUNCY WRIGHT.[1]

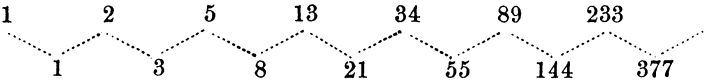
TAKE, by the finger and thumb of your right hand, hold of a spike of *Plantago major*, *Lepidium Virginicum*, or other flower-cluster with symmetrically crowded flowers, and with the finger and thumb of the left hand grasp it a little higher up, so as to include between the two hands a dozen or twenty buds on a piece of stem about equally tough from end to end. Twist the stem, and if it twists equally in all parts you will bring your buds into a small number of ranks, let us say 8. By twisting a little in the opposite direction you will get them into 5 ranks. Twist harder, and if your stem is tough enough to stand the twist you will bring them into two ranks. Turn back to 8 rows, and twist harder in that direction; you will fetch your buds into 3 rows. Then twist still harder in that direction, and if you have an old, tough, plaintain spike, you may get the seed-vessels all into one row before your stalk is twisted off.

[1] This article was prepared by Mr. Wright several years ago, at Professor Gray's suggestion. In its manuscript form it has been found of much interest and value to the botanical students in Harvard College. It is here reprinted without change from Mr. Wright's notes.

Thus by mechanical twisting, if the twist is equal in all parts of the stem, we get on one side of the natural position the number of rows 5 and 2, and on the other side 8, 3, and 1. Hence if we begin with the most twisted position and come toward the natural position, we get the numbers



Now these series of numbers indicate the approach towards the untwisted position. What would be the number of ranks in that theoretically perfect untwisted state? As both these series of numbers are increasing, that is, the number of ranks decreases as you twist either way, you may infer that in the untwisted state the number of ranks is prodigious or innumerable. Carrying on the series by adding zigzag as the lines are dotted, we should get



Hence we say that the slightest conceivable twist in one direction makes the number of ranks 377, a little more in that direction gives 144, 55, 21, 8, 3, 1, while the slightest twist in the opposite direction gives us 233, a little more 89, 34, 13, 5, 2, 1.

There is, however, a mystery in the space between 233 and 377, between twisting one way and twisting the other. Let us not seek to solve it by running the number of ranks up higher, to 610, 987, 1597, etc., but approach it in another way.

In the stem twisted one way, the angle between the leaves is $\frac{1}{2}$ the whole circumference, or $\frac{2}{5}$, or $\frac{5}{13}$, or $\frac{13}{34}$, etc.; with the stem twisted the other way the angle is $\frac{1}{3}$, or $\frac{3}{8}$, or $\frac{8}{21}$, or $\frac{21}{55}$ etc., the circumference. Let us set these in double rows: —

Twisted one way . . .	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{5}{13}$	$\frac{13}{34}$	$\frac{34}{89}$	$\frac{89}{233}$.	
Twisted the other way . .	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{8}{21}$	$\frac{21}{55}$	$\frac{55}{144}$	$\frac{144}{377}$.	

Or putting them in decimals we shall see how they converge towards the same value: —

Twisted one way . .	.5	.4	.38	.382	.38202
Twisted the other way .	.33	.375	.3809	.3818	.38194

Take the high fraction $\frac{1597}{4181}$ in the upper series and turn it into decimals, we get .38196603. If the leaves were at this angle they would form 4181 rows or ranks, and the least twist would produce the lower numbers. Let us now attempt to find some

simpler mode of representing these fractions .38196603 or $\frac{1597}{4181}$, which are the same.

Dividing both numerator and denominator of $\frac{1597}{4181}$ by 1597 will give $\frac{1}{2}+\frac{987}{1597}$; dividing each term of the last fraction by 987 gives us

$$\frac{1}{2+\frac{1}{1+\frac{610}{987}}}$$

and continuing the process gives us

$$\frac{1}{2+\frac{1}{1+\frac{1}{1+\frac{377}{610}}}}$$

equal to

$$\frac{1}{2+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{233}{377}}}}}$$

equal to

$$\frac{1}{2+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{144}{233}}}}}}$$

equal to

$$\frac{1}{2+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+}}}}}}}}}}$$ etc.

Now calling the fraction

$$\frac{1}{1+\frac{1}{1+\frac{1}{1}}}$$

continued indefinitely by the name of x, it is plain that the phyllotactic fractions beginning with $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{5}{13}$ continually approach nearer and nearer to the value $\frac{1}{2+x}$, or

$$\frac{1}{2+\frac{1}{1+x}}$$

or

$$\frac{1}{2+\frac{1}{1+\frac{1}{1+x}}}$$

and these values are alike. Putting the first two equal gives $\frac{1}{2+x}=\frac{1}{2+\frac{1}{1+x}}$

whence

$$x^2+x=1,\ x=-\tfrac{1}{2}+\tfrac{1}{2}\sqrt{5},\ \text{and}\ \frac{1}{2+x}=\tfrac{1}{2}\,(3-\sqrt{5}).$$

This expression, $\frac{1}{2}\,(3-\sqrt{5})$, is equal to .38196+, and expresses the exact ratio of the leaves in a theoretically untwisted stem when the number of rows is infinite. Other arrangements

are simply approximations to this (as though they aimed at this but got the stem twisted in growing), such as

$$1+\frac{1}{1}=\frac{1}{2} \qquad \frac{1}{2}+\frac{1}{1}=\frac{1}{3}$$

$$\frac{1}{2}+\frac{1}{1+\frac{1}{1}}=\frac{2}{5} \qquad \frac{1}{2}+\frac{1}{1+\frac{1}{1+\frac{1}{1}}}=\frac{3}{8}$$

TRACES OF AN AMERICAN AUTOCHTHON.

BY DR. C. C. ABBOTT.

WHEN in our rambles over the fields, in search of relics, we chance to find lying side by side some rough, rude implement and a delicate, artistically-wrought arrow-point, we are apt to merely glance at the former, and perhaps smile at so poor an effort at flint-chipping, while admiring the beauty of finish and excellence of workmanship displayed by the latter. But the unshapely implement has a history that, if not as eloquent as the legends of the red man, is far older, and calls up a shadowy vision of a still more distant time, when another people dwelt in this goodly land, and fashioned for its use these rude stone weapons that now alone are left to tell its story, and recall the time when this "great continent was occupied by a wide-spread though sparse population."

During the summer of 1872, having heard of the occurrence of Indian relics in a gravel bank then being removed, I carefully examined the face of the bluff, and succeeded in finding a single stone implement, and subsequently two others. These three specimens were described and figured soon after,[1] and I then expressed the opinion that, "had but a single specimen been found, we might reasonably, perhaps, have applied to it the doctrine of chances, and maintained that it was merely a freak of nature; but the occurrence of three specimens so near each other effectually disposes of the justice of such an opinion, and we must admit the antiquity of American man to be greater than the advent of the so-called Indian."

I have lately succeeded in finding a few specimens of relics, in strata of river drift, similar to those figured in the NATURALIST, but higher up in the series; thus apparently connecting them with the rude forms found near and occasionally on the surface, which I formerly believed to be the forerunners of the later

[1] The American Naturalist, vii. 204.